Hong Kong Mathematics Olympiad (1995 – 96)
Final Event 6 (Group)
香港數學競賽 (1995 – 96)
決賽項目 6 (團體)

(i) The number of eggs in a basket was a. Eggs were given out in three rounds. In the first round half of the eggs plus half an egg were given out. In the second round, half of the remaining eggs plus half an egg were given out. In the third round, again, half of the remaining eggs plus half an egg were given out. The basket then became empty. Find a.

一籃子雞蛋的數目為 a，分三輪派發。第一輪派出一半另半枚，第二輪派出剩下的一半另半枚，第三輪又派出剩下的一半另半枚。籃子中的雞蛋便全部派光，求 a。

$a =$

(ii) If $p-q=2$; $p-r=1$ and $b=(r-q)\left[(p-q)^2+(p-q)(p-r)+(p-r)^2\right]$. Find the value of b.

$b =$

若 $p-q=2$；$p-r=1$ 及 $b=(r-q)\left[(p-q)^2+(p-q)(p-r)+(p-r)^2\right]$，求 b 的值。

(iii) If n is a positive integer, $m^{2n}=2$ and $c=2m^{6n}-4$, find the value of c.

$c =$

若 n 是一正整數，$m^{2n}=2$ 及 $c=2m^{6n}-4$，求 c 的值。

(iv) If r, s, t, u are positive integers and $r^5=s^4$, $t^3=u^2$, $t-r=19$ and $d=u-s$, find the value of d.

$d =$

若 r, s, t, u 是正整數及 $r^5=s^4$，$t^3=u^2$，$t-r=19$ 及 $d=u-s$，求 d 的值。

Hong Kong Mathematics Olympiad (1995 – 96)

Final Event 7 (Group)

香港數學競賽 (1995 – 96)

決賽項目 7 (團體)

(i) If the two roots of the equation $ax^2 - mx + 1996 = 0$ are primes, find the value of a.

$a =$

若方程 $ax^2 - mx + 1996 = 0$ 的兩個根是質數，求 a 的值。

(ii) A six-digit figure $111aaa$ is the product of two consecutive positive integers b and $b + 1$, find the value of b.

$b =$

六位數 $111aaa$ 是兩個連續正整數 b 和 $b + 1$ 之積，求 b 的值。

(iii) If p, q, r are non-zero real numbers; $p^2 + q^2 + r^2 = 1$,

$p\left(\frac{1}{q}+\frac{1}{r}\right)+q\left(\frac{1}{r}+\frac{1}{p}\right)+r\left(\frac{1}{p}+\frac{1}{q}\right)+3=0$ and $c = p + q + r$, find the value of c.

$c =$

若 p, q, r 是非零實數，$p^2 + q^2 + r^2 = 1$，

$p\left(\frac{1}{q}+\frac{1}{r}\right)+q\left(\frac{1}{r}+\frac{1}{p}\right)+r\left(\frac{1}{p}+\frac{1}{q}\right)+3=0$ 及 $c = p + q + r$,，求 c 的最大值。

(iv) If the unit digit of 7^{14} is d, find the value of d.

$d =$

若 7^{14} 之個位是 d，求 d 的值。

Hong Kong Mathematics Olympiad (1995 – 96)

Final Event 8 (Group)

香港數學競賽 (1995 – 96)

決賽項目 8 (團體)

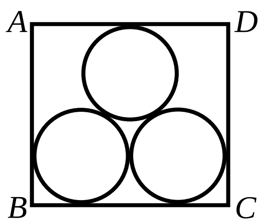
In this question, all unnamed circles are unit circles.

在本題內，所有不命名的圓皆為單位圓。

(i) If the area of the rectangle $ABCD$ is $a+4\sqrt{3}$, find the value of a.

$a =$

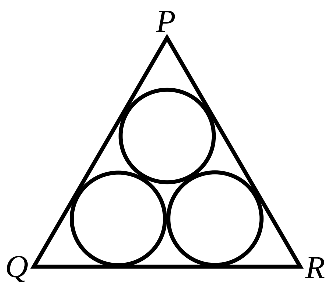
若矩形 $ABCD$ 的面積是 $a+4\sqrt{3}$，求 a 的值。



(ii) If the area of the equilateral triangle PQR is $6+b\sqrt{3}$, find the value of b.

$b =$

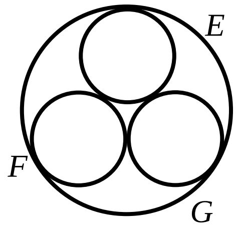
若等邊三角形 PQR 的面積是 $6+b\sqrt{3}$，求 b 的值。



(iii) If the area of the circle EFG is $\frac{(c+4\sqrt{3})\pi}{3}$, find the value of c.

$c =$

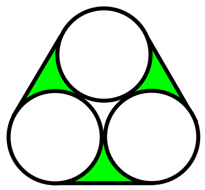
若圓 EFG 的面積是 $\frac{(c+4\sqrt{3})\pi}{3}$，求 c 的值。



(iv) If all the straight lines in the diagram below are common tangents to the two circles, and the area of the shaded part is $6+d\pi$, find the value of d.

$d =$

若下圖所有直線皆為兩個圓的公切線，且陰影部分的面積是 $6+d\pi$，求 d 的值。



Hong Kong Mathematics Olympiad (1995 – 96)
Final Event 9 (Group)
香港數學競賽 (1995 – 96)
決賽項目 9 (團體)

(i) If $(1995)^a+(1996)^a+(1997)^a$ is divisible by 10, find the least possible integral value of a.

$a =$

若 $(1995)^a+(1996)^a+(1997)^a$ 能被 10 整除，求 a 的最小可能整數值。

(ii) If the expression $\left(x^2+y^2\right)^2 \le b\left(x^4+y^4\right)$ holds for all values of x and y, find the least possible integral value of b.

$b =$

若 $\left(x^2+y^2\right)^2 \le b\left(x^4+y^4\right)$ 對任意實數 x 和 y 都成立，求 b 的最小可能整數值。

(iii) If $c=1996\times19971997-1995\times19961996$, find the value of c.

$c =$

若 $c=1996\times19971997-1995\times19961996$，求 c 的值。

(iv) Find the sum d where

$$d=\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{60}\right)+\left(\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\cdots+\frac{2}{60}\right)+\left(\frac{3}{4}+\frac{3}{5}+\cdots+\frac{3}{60}\right)+\cdots+\left(\frac{58}{59}+\frac{58}{60}\right)+\frac{59}{60}$$

若

$$d=\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{60}\right)+\left(\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\cdots+\frac{2}{60}\right)+\left(\frac{3}{4}+\frac{3}{5}+\cdots+\frac{3}{60}\right)+\cdots+\left(\frac{58}{59}+\frac{58}{60}\right)+\frac{59}{60}$$

求 d 的值。

$d =$

Hong Kong Mathematics Olympiad (1995 – 96)
Final Event 10 (Group)
香港數學競賽 (1995 – 96)
決賽項目 10 (團體)

(i) It is given that 已知

$$3\times4\times5\times6=19^2-1$$
$$4\times5\times6\times7=29^2-1$$
$$5\times6\times7\times8=41^2-1$$
$$6\times7\times8\times9=55^2-1$$

If $a^2=1000\times1001\times1002\times1003+1$, find the value of a.

$a=$

若 $a^2=1000\times1001\times1002\times1003+1$，求 a 的值。

(ii) Let $f(x)=x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1$. When $f(x^{10})$ is divided by $f(x)$, the remainder is b. Find the value of b.

$b=$

設 $f(x)=x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1$。當 $f(x^{10})$ 除以 $f(x)$，餘數是 b。求 b 的值。

(iii) The fraction $\frac{p}{q}$ is in its simplest form. If $\frac{7}{10}<\frac{p}{q}<\frac{11}{15}$ where q is the smallest possible positive integer and $c=pq$. Find the value of c.

$c=$

分數 $\frac{p}{q}$ 已化成最簡形式。若 $\frac{7}{10}<\frac{p}{q}<\frac{11}{15}$，其中 q 是最小可能正整數，且 $c=pq$，求 c 的值。

(iv) A positive integer d when divided by 7 will have 1 as its remainder; when divided by 5 will have 2 as its remainder and when divided by 3 will have 2 as its remainder. Find the least possible value of d.

$d=$

若正整數 d 除以 7，餘數是 1；除以 5 餘數是 2；除以 3 餘數是 2。求 d 的最小可能值。